experiment,[16] and the branching ratio to be about 0.96×10^{-3}, also consistent with experiment.[16]

Finally, we can make use of Table I and the above determination of the axial-vector coupling constants to derive the strong meson-baryon pseudoscalar coupling constants. These are summarized in Table II.

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Selection Rules and Some Relations in W_3 Symmetry

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In the limit of W_3 symmetry proposed by Schwinger, selection rules and some relations among partial cross sections are derived through elementary calculations. One of the main results is that $\pi+N\to\Sigma+n$ pseudoscalar mesons, $\bar{K}+N\to\Sigma+n$ pseudoscalar mesons, and $\bar{K}+N\to\Xi+n$ pseudoscalar mesons are forbidden as well as their crossed reactions. Many of the selection rules and the relations derived here are badly violated at least in low-energy region. However, they should be tested with experimental data at much higher energies than those available at present.

1. INTRODUCTION

RECENTLY, Schwinger[1] proposed $U(3)\times U(3)$ symmetry or W_3 symmetry, as he called it, as an alternative to SU(3) symmetry in the strong interactions. In this paper we derived selection rules and some other relations among partial cross sections, which are readily susceptible to experimental test. These are valid in the limit of complete symmetry.

In Schwinger's scheme there are the two fundamental triplets, one is ψ_μ with baryon number 1 and the other is V_α with baryon number 2. The baryons are constructed from $\bar{\psi}$ and V, while the pseudoscalar and the vector mesons are from ψ and $\bar{\psi}$. The nine baryons are represented as the following matrix:

$$B=\bar{\psi}^\mu V_\alpha=\begin{bmatrix} (1/\sqrt{6})\Lambda+(1/\sqrt{2})\Sigma^0+(1/\sqrt{3})Y & \Sigma^- & n \\ \Sigma^+ & (1/\sqrt{6})\Lambda-(1/\sqrt{2})\Sigma^0+(1/\sqrt{3})Y & p \\ \Xi^0 & \Xi^- & -(\frac{2}{3})^{1/2}\Lambda+(1/\sqrt{3})Y \end{bmatrix}, \quad (1.1)$$

where Y is the ninth baryon. As was pointed out by Schwinger, the Y_0^* (1405 MeV) is the promising candidate for the ninth baryon, if its spin and parity turn out to be $\frac{1}{2}^+$. It should be noted here that the row and the column refer to the indices of $\bar{\psi}$ and V, respectively, in the matrix given above.[2] The assignment of the mesons is exactly the same with that of SU(3) symmetry,

$$P=\bar{\psi}^\mu\psi_\nu=\begin{bmatrix} (1/\sqrt{6})\eta+(1/\sqrt{2})\pi^0 & \pi^- & K^0 \\ \pi^+ & (1/\sqrt{6})\eta-(1/\sqrt{2})\pi^0 & K^+ \\ \bar{K}^0 & K^- & -(\frac{2}{3})^{1/2}\eta \end{bmatrix}, \quad (1.2)$$

for the pseudoscalar mesons and the similar matrix for the vector mesons. In contrast with the case of the baryons both the row and the column are simultaneously transformed under the same U(3).

[1] J. Schwinger, Phys. Rev. Letters 12, 237 (1964).

[2] There is another assignment which is completely equivalent from the group-theoretical viewpoint. We can obtain it by exchanging the row and the column in Eq. (1.1). In this assignment, however, the $NN\pi$ coupling turns out to be zero in the W_3-symmetric limit.

2. INVARIANT AMPLITUDES OF FOUR-LEG DIAGRAMS

Let us construct invariant forms from the field operators of two baryons and two mesons. As is easily seen, the invariant amplitudes are written as

$$\begin{aligned} M=f\,\mathrm{Tr}[\{B\bar{B}-(1/3)\,\mathrm{Tr}(B\bar{B})\}(P_1P_2+P_2P_1)] \\ +g\,\mathrm{Tr}[\{B\bar{B}-(1/3)\,\mathrm{Tr}(B\bar{B})\}(P_1P_2-P_2P_1)] \\ +e\,\mathrm{Tr}(B\bar{B})\,\mathrm{Tr}(P_1P_2). \quad (2.1) \end{aligned}$$

Indeed, it is a direct consequence of the following decomposition of the product representations:

$$B\bar{B}=(\bar{3}\times 3,\, 3\times\bar{3}) = (8,8)+(8,1)+(1,8)+(1,1), \quad (2.2)$$

$$P_1P_2=(8\times 8,\, 1) = (27,1)+(10,1)+(\bar{10},1)+(8_A,1) + (8_S,1)+(1,1), \quad (2.3)$$

where the first entry in the bracket refers to the representations of U(3) associated with ψ and $\bar{\psi}$, while the second entry refers to those associated with V and $\bar{V}$. The relevant irreducible representations are $(8_A,1)$, $(8_S,1)$, and $(1,1)$ for P_1P_2. Since the last term is responsible only for elastic scatterings and their crossed reactions, we have two parameters for the boson-nucleon reactions. Thus we are led to the much stronger restrictions upon two-body reactions in W_3 symmetry than in SU(3).

3. TWO-BODY PROCESSES

First, we enumerate the consequences in the boson-nucleon scatterings. In order to avoid the complicated numerical coefficients due to mass differences, the relations are written in terms of invariant cross sections, which are usually taken as

$$\bar{\sigma}=(q_i/q_f)\sigma_{\text{obs}}, \quad (3.1)$$

where q_i and q_f are the c.m. momenta of the initial and final states, respectively. We have tabulated them in Table I.

One of the striking characteristics is that the associated production $\pi^-p\rightarrow(K\Sigma)^0$ is forbidden as well as $\pi^+p\rightarrow K^+\Sigma^+$. This mode has been observed almost as frequently as $\pi^-p\rightarrow K^0\Lambda$[3–6] from near the thresholds up to the region of a few BeV. Another characteristic is that some of the strangeness-exchange processes, $K^-p\rightarrow(\pi\Sigma)^0$, $K^-p\rightarrow(K\Xi)^0$, $K^-n\rightarrow(\pi\Sigma)^-$, and $K^-n\rightarrow K^0\Xi^-$ are forbidden. Existence of the first and the second processes has been firmly established in many experiments.[7–9] They do not appear to be suppressed as compared with the allowed process $K^-p\rightarrow\pi^0\Lambda$. On the contrary, $K^-p\rightarrow(\pi\Sigma)^0$ is dominant over $K^-p\rightarrow\pi^0\Lambda$ near the threshold.

We have a simple relation between $\pi^-p\rightarrow K^0\Lambda$ and $K^-p\rightarrow\bar{K}^0n$, namely,

$$\bar{\sigma}(\pi^-p\rightarrow K^0\Lambda)=(2/3)\bar{\sigma}(K^-p\rightarrow\bar{K}^0n). \quad (3.2)$$

In addition, we can derive many other sum rules of transition amplitudes, or triangular relations among partial cross sections, which can be easily written down from the entries in Table I.

Next, consider elastic scatterings. The resulting relations are

$$\begin{aligned}\bar{\sigma}(\pi^+p\rightarrow\pi^+p)&=\bar{\sigma}(K^+p\rightarrow K^+p),\\ \bar{\sigma}(\pi^-p\rightarrow\pi^-p)&=\bar{\sigma}(K^-p\rightarrow K^-p),\\ \bar{\sigma}(K^+n\rightarrow K^+n)&=\bar{\sigma}(K^-n\rightarrow K^-n),\end{aligned} \quad (3.3)$$

as well as many triangular relations derived from Tables I and II. These relations are badly violated, at least in the region of the baryon isobar formations. They appear, however, to approach each other at high energy.[10,11] Optimistically speaking, this fact may suggest that all the relations in this paper might turn out to be valid at sufficiently high energy. Nevertheless, it should be remembered that Eq. (3.3) can be quite generally derived in the high-energy limit from simple considerations based on some reasonable assumptions.[12]

TABLE I. Reaction amplitudes of the boson-nucleon reactions.[a] F and G are invariant amplitudes associated with the $(8_S,1)$ and $(8_A,1)$ representations in Eqs. (2.2) and (2.3). Those amplitudes which can be uniquely determined from charge independence are omitted throughout all the tables in the present papers.

Processes	Reaction amplitudes
$\pi^-p\rightarrow\pi^0n$	$-\sqrt{2}G$
$\rightarrow\eta n$	$(2/3)^{1/2}F$
$\rightarrow K^0\Lambda$	$-(2/3)^{1/2}F+(2/3)^{1/2}G$
$\rightarrow K^0Y$	$(1/3)^{1/2}F-(1/3)^{1/2}G$
$K^-p\rightarrow\bar{K}^0n$	$F-G$
$\rightarrow\pi^0\Lambda$	$(1/3)^{1/2}F+(1/3)^{1/2}G$
$\rightarrow\eta\Lambda$	$(1/3)F-G$
$\rightarrow\pi^0Y$	$-(1/6)^{1/2}F-(1/6)^{1/2}G$
$\rightarrow\eta Y$	$-1/(3/\sqrt{2})F+(1/2)^{1/2}G$
$\bar{K}^0p\rightarrow\pi^+\Lambda$	$-(2/3)^{1/2}F-(2/3)^{1/2}G$
$\rightarrow\pi^+Y$	$(1/3)^{1/2}F+(1/3)^{1/2}G$

[a] $\pi N\rightarrow\bar{K}\Sigma$, $KN\rightarrow\pi\Sigma$ and $\bar{K}N\rightarrow K\Xi$ are all forbidden.

TABLE II. Elastic amplitudes of the boson-nucleon scatterings. E is an invariant amplitude associated with the identity representation in Eqs. (2.2) and (2.3).

Processes	Scattering amplitudes
$\pi^+p\rightarrow\pi^+p$	$(1/3)F+G+E$
$\pi^-p\rightarrow\pi^-p$	$(1/3)F-G+E$
$K^+p\rightarrow K^+p$	$(1/3)F+G+E$
$K^+n\rightarrow K^+n$	$-(2/3)F\ +\ E$
$K^-p\rightarrow K^-p$	$(1/3)F-G+E$
$K^-n\rightarrow K^-n$	$-(2/3)F\ +\ E$

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[12] M. Suzuki, Phys. Letters **6**, 204 (1963).

TABLE III. Reaction amplitudes of the proton-antibaryon annihilations.[a]

Processes	Reaction amplitudes
$p\bar{p} \to \pi^+\pi^-$	$(1/3)\bar{F}+\bar{G}+\bar{E}$
$\to K^+K^-$	$(1/3)\bar{F}+\bar{G}+\bar{E}$
$\to K^0\bar{K}^0$	$-(2/3)\bar{F}+\bar{E}$
$\to \pi^0\eta$	$-(1/\sqrt{3})\bar{F}$
$\to \eta\eta$	$-(1/3)\bar{F}+\bar{E}$
$p\bar{n} \to \pi^+\pi^0$	$-\sqrt{2}\bar{G}$
$\to \pi^+\eta$	$(2/3)^{1/2}\bar{F}$
$\to K^+\bar{K}^0$	$\bar{F}+\bar{G}$
$p\bar{\Lambda} \to \pi^+K^0$	$-(2/3)^{1/2}\bar{F}-(2/3)^{1/2}\bar{G}$
$\to \eta K^+$	$(1/3)\bar{F}-\bar{G}$

[a] $p\bar{\Sigma} \to$ pseudoscalar mesons and $p\bar{\Xi} \to$ pseudoscalar mesons are all forbidden.

Finally let us investigate proton-antibaryon annihilations.[13] In addition to many triangular relations among partial cross sections (see Table III), we have

$$\bar{\sigma}(p\bar{p} \to \pi^+\pi^-) = \bar{\sigma}(p\bar{p} \to K^+K^-),$$
$$\bar{\sigma}(p\bar{n} \to K^+\bar{K}^0) = (3/2)\bar{\sigma}(p\bar{\Lambda} \to \pi^+K^0). \quad (3.4)$$

If we confine ourselves to the annihilations at rest (s-wave capture), $\bar{F}=\bar{E}=0$ follows from the Bose statistics of two pseudoscalar mesons, since $\bar{F}$ and $\bar{E}$ are associated with $(8_S,1)$ and $(1,1)$. Then

$$\bar{\sigma}(p\bar{p} \to \pi^+\pi^-) : \bar{\sigma}(p\bar{p} \to K^+K^-) : \bar{\sigma}(p\bar{n} \to \pi^+\pi^0) :$$
$$\bar{\sigma}(p\bar{n} \to K^+\bar{K}^0) : \bar{\sigma}(p\bar{\Lambda} \to \pi^+K^0) : \bar{\sigma}(p\bar{\Lambda} \to \eta K^+)$$
$$= 1:1:2:1:(2/3):1, \quad (3.5)$$

$$\bar{\sigma}(p\bar{p} \to K^0\bar{K}^0) = \bar{\sigma}(p\bar{p} \to \pi^0\eta) = \bar{\sigma}(p\bar{p} \to \eta\eta) = 0. \quad (3.6)$$

As for $p\bar{\Sigma}$ and $p\bar{\Xi}$ annihilations, not only two-meson annihilations but also annihilations into arbitrary number of mesons are forbidden. That is, both $p\bar{\Sigma}$ and $p\bar{\Xi}$ reactions are always accompanied by the baryon-antibaryon pairs. If these rules held accurately, $\bar{\Sigma}$ and $\bar{\Xi}$ would look metastable in nuclear matter. These selection rules arise directly from the fact that the 3×3 matrix $B\bar{B}$ does not contain $\bar{\Sigma}N$ or $\bar{\Xi}N$ in its elements.

[13] R. Armenteros, L. Montanet, D. R. O. Morrison, S. Nilsson *et al.*, in Ref. 3, p. 351.

4. REMARKS ON INELASTIC PROCESSES

We should like to discuss here many-particle production processes.[8,9,14] We can directly extend the selection rules given above to the processes

$$\pi+N \to \Sigma+n \text{ pseudoscalar mesons } (n\geqq 1), \quad (4.1)$$

and

$$\bar{K}+N \to \Sigma(\Xi)+n \text{ pseudoscalar mesons } (n\geqq 1). \quad (4.2)$$

They are forbidden in the limit of W_3 symmetry. Just as was remarked at the end of the preceding section, it is again due to the fact that $B\bar{B}$ contains no $\bar{\Sigma}N$ or $\bar{\Xi}N$.

Whereas the two-body reaction cross sections rapidly decrease as energy increases, in accord with the conjecture by Okun and Pomeranchuk,[15] the cross sections of the inelastic processes listed here will decrease much more gradually. It is vaguely expected that a symmetry scheme can be better tested at high energy, since symmetry-breaking effects play a smaller role there. From this viewpoint the selection rules for the inelastic processes are the most sensible tests for W_3 symmetry. If they become sufficiently small in comparison with the allowed processes, namely, charge-exchange and Λ production processes, it will be a strong support for W_3 symmetry as a basic internal symmetry scheme. This is still not the case in the energy region covered with the present experimental data (up to about 10 BeV[10]).

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Note added in proof. After submission of this work, A. Pais published a Letter in which he independently pointed out the selection rules about inelastic Σ and Ξ productions [A. Pais, Phys. Rev. Letters **12**, 632 (1964)]. The present paper contains in addition all the consequences readily compared with experiment in two-body processes.

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